Probabilistic Fatigue Design

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**P•FAT—Probabilistic Fatigue Assessment Tool**

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Finite element post-processor
Introduction

The defect size distribution is related to the manufacturing process. Typical volume defects are

- Non-metallic inclusions
- Pores
- Shrinkage cavities

The list of surface defects include

- Machining marks
- Corrosion pits
- Non-metallic inclusions or pores located close to or at the surface
Scanning electron microscopy images

a)

b)
Inspection methods

- Optical microscopy
- Non-destructive testing
- Inclusion concentration method
- Chemical analysis
- Fracture methods
- Oxygen determination
- Spark emission

H.V. Atkinson and G. Shi
Characterization of inclusions in clean steels: a review including the statistics of extremes methods.

Approaches for estimating the sizes of large defects

- Block maximum method
- Peak over threshold method

C.W. Anderson, J. de Maře and H. Rootzèn
Methods for estimating the sizes of large inclusions in clean steels.

Approaches for estimating the sizes of large defects

Block maximum method

- The total inspection region is divided into $k$ equally sized polished cross-regions
- $k$ observations of maximum defect sizes, $a_{\text{max}1}, \ldots, a_{\text{max}k}$
- The generalised extreme value (GEV) distribution is fitted to these data
- The GEV distribution combines the Gumbel, Fréchet and the reversed Weibull distributions into a single distribution
Approaches for estimating the sizes of large defects

Peak over threshold method

- All defects above a certain high threshold are considered
- The result is a set of $i$ observations, $a_1, \ldots, a_i$
- The overshoot of the defect sizes above the threshold are fitted to a generalised Pareto distribution
Number, position and size of defects

**Number of defects**

- The number of defects in each finite element is obtained by ‘drawing’ from a Poisson distribution.
Number, position and size of defects

Location of defects

- The location of each defect in an element is found by drawing from a uniform distribution
Number, position and size of defects

Size of defects

- The defect size is obtained by drawing from a (i) generalised extreme value distribution or a (ii) generalised Pareto distribution

Graph showing the relationship between defect size and some statistical measure.
Fatigue limit and condition for propagation of defects

Kitagawa-Takahashi diagram

\[ \Delta \sigma = \frac{\Delta \sigma_A}{\sqrt{1 + a/a'}} \]

\[ a' = \frac{1}{\pi} \left( \frac{\Delta K_{th}}{F \Delta \sigma_a} \right)^2 \]
Fatigue limit and condition for propagation of defects

\[
\Delta K = \frac{\Delta K_{\text{th}}}{\sqrt{1 + a'/a}}
\]

Kitagawa-Takahashi diagram
Fatigue limit and condition for propagation of defects

Kitagawa-Takahashi diagram

\[
\left( \frac{\Delta K}{\Delta K_{th}} \right)^2 + \left( \frac{\Delta \sigma}{\Delta \sigma_A} \right)^2 = 1
\]
Fatigue limit and condition for propagation of defects

Critical defect size

- The smallest defect initiating failure in $\Delta V$ is denoted by $a_{\text{crit}}$

$$a_{\text{crit}} = a' \left[ \left( \frac{\Delta \sigma_A}{\Delta \sigma} \right)^2 - 1 \right]$$
The probability of survival of a component of volume $V$ equals the product of the probabilities of survival of all the volume elements

$$P_{s,V} = \exp \left\{ - \sum_{i=1}^{V/\Delta V} \left[ 1 + \xi' \left( \frac{a_{\text{crit}} - a_0^*}{a_0} \right) \right]^{-1/\xi'} \frac{\Delta V_i}{V_0} \right\}$$

$$P_{f,V} = 1 - P_{s,V}$$
Probability of fatigue failure

Block maximum method

When the maximum defect size follows a two-parameter Fréchet distribution and using a power-law relationship between the defect size and the applied stress amplitude $\sigma_a$, we have:

$$P_{f,V} = 1 - \exp\left[-\int_{V} \left(\frac{\sigma_a}{\sigma^*_{A0}}\right)^{b_\sigma} \frac{dV}{V_0}\right]$$

i.e., a two-parameter Weibull distribution
Peak over threshold method

\[ z_0(a_{\text{th}}) \text{ denotes the expected number of defects with sizes greater than the threshold } a_{\text{th}} \text{ per unit volume of the material} \]

\[
P_{f,V} = 1 - \exp \left\{ - \frac{V}{\Delta V} \sum_{i=1}^{V/\Delta V} z_0(a_{\text{th}}) \left[ 1 + \xi' \left( \frac{a_{\text{crit}} - a_{\text{th}}}{\bar{a}_0} \right) \right]^{-1/\xi'} \Delta V_i \right\}
\]
Fatigue life distribution of notched plate

a) $t = 100 \text{ mm}$

b) $w = 140 \text{ mm}$

$d = \rho$

$\rho = 20 \text{ mm}$

$h = 500 \text{ mm}$
Fatigue life distribution of notched plate

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fatigue_distribution.png}
\caption{Normalized fatigue life distribution of a notched plate.}
\end{figure}