Preview:
The fatigue limit: An analytical solution to a Monte Carlo problem

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Motivation & Outline

Probabilistic models

Weibull type models:
- Black box
- Pragmatic
- Simple Application

Hybrid models:
- General
- Simple Application

Monte Carlo:
- Explicit – White box - Physical
- Very specific – No generality
- Input issues
- Computer Simulations

Hybrid models: Best of both worlds?
Hybrid models: Cost of generality?

- Are significant features and processes being neglected in the name of generality?

My opinion: Probably...
Motivation & Outline

Motivation:
Discard excessive generality
Extend the limits of hybrid models

Our model:
Analytical solution to a Monte Carlo problem
Analytical solution to a Monte Carlo problem

“It is more important to have beauty in one's equations than to have them fit experiment.”

Paul Dirac
Definition of the Monte Carlo Problem

General hypothesis:

The fatigue limit is a process controlled by the most severe defect present in a component.
Definition of the Monte Carlo Problem – cont’d

- Materials contain discrete defects with random sizes and spatial location.
- The defects are «virtually» spherical in shape.
- The defect sizes are log-normal distributed.
- The defect-stress relationship is defined by the $\sqrt{\text{area}}$ model (Murakami).
Definition of the Monte Carlo Problem – cont’d

Log-Normal distribution: Why?

- Physically sound
- Mathematically attractive
- Often used to describe defects
Definition of the Monte Carlo Problem – cont’d

The correct approach should be:

Log-Normal distribution: Why not?
Definition of the Monte Carlo Problem – cont’d

“Surface” defects!
The Model

Probability of failure

\[ F_{FL}(S_\Lambda) = 1 - \exp \left\{ - \left( \frac{\rho}{2} \int_V \text{erfc} \left( \frac{\mu_S - \ln s}{\sqrt{2} \nu_S} \right) \, dV + \frac{\rho S}{2} \int_M \int_{-1}^1 \text{erfc} \left( \frac{\nu_S - \frac{h(u)}{6} + \ln \frac{1.43}{1.56}}{\sqrt{2} \nu_S} - \ln s \right) \, du \, dM \right) \right\} \]

Model parameters

Stochastic behavior – Estimation from data:
\( \mu_S, \nu_S \)

Material properties – Obtainable / Guessable:
\( H_V, \rho \)

Load case - Known:
\( R, \alpha \)

Only 2 fitting parameters
Example Application

Böhm data: 30CrNiMo8 steel
3 smooth and 8 notched specimen
Example Application

Material properties
\( R_{p0.2} = 770 - 828 \text{MPa} \)
\( R_m = 898 - 966 \text{MPa} \)
Cyclic softening

Model parameters:
\( H_V = 290 \)
\( \rho = 25 \text{ defects per } mm^3 \)
\( R = -1 \)

\( \mu_S \) and \( \nu_S \) estimated from one of the smooth specimen
(Fully reversed, tension-compression)
Results

Success criteria: less than 15% error

Training specimen

![Graph showing SA - The fatigue limit with Predicted (MPa) on the y-axis and Experimental (MPa) on the x-axis. The graph includes a trend line and data points labeled "Training".]
Results

Success criteria: less than 15% error

Training specimen + smooth specimen
Results

Success criteria: less than 15% error

Training specimen + smooth specimen + notched specimen
Results

Success criteria: less than 15% error

Training specimen + smooth specimen + notched specimen
# Failed predictions?

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<th>Specimen</th>
<th>Kt</th>
<th>Relative stress gradient</th>
<th>Exp. $S_A$</th>
<th>Exp. local peak</th>
<th>Predicted $S_A$</th>
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Failed when it is suppose to fail!
Conclusion

• A robust and transparent probabilistic model is proposed

• The model is physically based

• Initial tests of performance are very promising