FRACTURE

fracture
Outline of Presentation

- Critical CTOA (or CTOD) and finite-element analyses of fracture for laboratory specimens and cracked structural configurations
- Validation of Two-Parameter Fracture Criterion (TPFC) using critical CTOA and finite-element analyses
- Equivalent KR-Curve from TPFC failure analyses
- Derivation of TPFC from Notch-Strength Analyses
Early Research Lead to Development of CTOA

ARIJ DE KONING
1942–2007
FATIGUE FAILURE IN MODERN AIRCRAFT

Widespread Fatigue Damage (WFD)

Aloha Airlines Boeing 737
April 28, 1988
CTOA and \( \delta_5 \)–R Curve Fracture Parameters

Tearing cracks are sharp and are “arrow” shaped.

After 5-mm crack growth
C.T. Sun, Purdue
Mild steel (\( \sigma_{ys} = 430 \) MPa)
B = 2 mm
ASTM and ISO Fracture Standards

- **ASTM E-2472-06** – Standard Test Method for Determination of Resistance to Stable Crack Extension under Low-Constraint Conditions


Two fracture parameters are covered:

- CTOA (constant between $\Delta c_{\text{min}}$ and $\Delta c_{\text{max}}$)
- $\delta_5 - \Delta c$ Resistance curve (unique for $\Delta c < \Delta c_{\text{max}}$) for compact C(T) and middle-crack tension M(T) specimens under specific crack-configuration restraints (such as $c/B > 4$)
Laboratory Fracture Specimens

(a) Compact

(b) Middle-crack  (c) Three-hole-crack
CTOA Fracture Test System

- Load-controlled pre-cracking ($c_i/W = 0.4$)
- Incremental displacement controlled tearing
- Load-line displacement, load measurements, CTOA, $\delta_5$, $\Delta c$
CTOA Measurements on Low-Strength Steel

C. T. Sun, Purdue University
Steel: $\sigma_{ys} = 430 \text{ MPa}$

(a) Before crack initiation

(b) 0.2-mm crack growth

(c) 5-mm crack growth
CTOA and Crack-Extension Measurements

Before Crack Extension

After Crack Extension

- CTOA measured from frame prior to crack extension
- 6-10 angle measurements for each crack extension value
- Digital Image Correlation used to measure CTOA for some specimens to validate the optical micro-scope (OM) technique
Measured Critical CTOA Values on “Thin” Lab Specimens

\[ \Delta c_{\text{min}} \]

\[ \psi_C, \text{ degs.} \]

\[ 2024-T3 \ B = 2.3 \text{ mm} \]

- C(T) W = 152 mm
- O  M(T) W = 76 mm
- □ M(T) W = 305 mm
- △ M(T) W = 610 mm

\[ \psi_C = 5.8 \text{ degs.} \]

\[ c_0 / B = 5 \text{ to } 50 \]
Measured Critical CTOA Values on “Thick” Lab Specimens

2024-T351
B = 25.4 mm
C(T) W = 203 mm
c_o/B = 3.2

 hindi
Two-Dimensional Finite-Element Fracture Simulations

What’s Wrong with This Chart?

- Shih et al. (1979) A533B
- Newman et al. (1989) A533B
- Brocks and Yuan (1989) 2000-series
- James et al. (2003) 2024-T351
- Kanninen et al. (1979) 2219-T87

CTOA $\Psi_c$, degs.

Surface crack extension, $\Delta c_s$, mm
Two-Dimensional Finite-Element Fracture Simulations

• **Three-Dimensional Fracture Process** – Two-dimensional finite-element modeling (either plane-stress or plane-strain) *does not* capture the constraint variations during the fracture process.

• **Crack Tunneling** – Load-against-crack extension data were measured on the free surface and used in a generation mode to calculate CTOA, but many (thinner gage) materials severely tunnel in the interior.

• **Crack Blunting** – Initial cracks are fatigue pre-cracked leaving a smaller amount of residual-plastic deformations than during the fracture (stable tearing) process.
7075-T651  B = 12.7 mm

Participant rank, method and number in predicting failure:

(2) K-R Curve [15]
(3) CTOD (or CTOA) finite-element fracture analysis [17]
(4) K-R Curve [14]
(5) …
ASTM Fracture Round Robin (STP-896, 1985) (2)

Participant rank, method and number in predicting failure:

1. Two-Parameter Fracture Criterion (TPFC) [4]
2. CTOD (or CTOA) finite-element fracture analysis [17]
3. Modified Two-Parameter Fracture Criterion (TPFC) [5]
5. …

2024-T351  B = 12.7 mm
ASTM Fracture Round Robin (STP-896, 1985) (3)

304 Stainless Steel  B = 12.7 mm

Participant rank, method and number in predicting failure:

(1) Limit-load criterion [15]
(2) CTOD (or CTOA) finite-element fracture analysis [17]
(3) Limit-load criterion [14]
(4) Two-Parameter Fracture Criterion (TPFC) [4]
(5) …
CTOA Fracture Criterion Accounts for Pre-Cracking Effects

![Graph showing the relationship between applied stress and surface crack extension. The equation $S_n = \sigma_{ys}$ is shown, along with data points for different materials and conditions.]

- **2024-T3** B = 1.8 mm
- M(T): W = 300 mm; $c_o = 50.8$ mm
- $\Psi_c = 6.1$ deg.

**Test** Analysis Pre-cracking stress, MPa
- ○ 28
- ■ 155

**Surface crack extension, $\Delta c_s$, mm**

**Applied stress, S, MPa**

- [Graph axes and data points]
Plane-Strain Core in Plane-Stress Finite-Element Model

Finite-element codes:
- ZIP2D
- STAGS
- FEA – C.T. Sun (Purdue Univ.)
CTOA Fracture Simulations Need Constraint Variations

Test (anti-buckling guides)

Plane stress ($\Psi_0 = 4.7$ deg.)

Plane-strain core

ZIP2D Analyses:

- Plane-strain core ($\Psi_0 = 4.7$ deg.; $h_0 = 1.9$ mm)
- Plane strain ($\Psi_0 = 4.7$ deg.)

2219-T87

- $B = 2.54$ mm
- $2c_0/W = 1/3$
Three-Dimensional Modeling of Tunneling and Tearing (ZIP3D)

Dawicke et al, 1997

2024-T3 TL-Orientation Flat fracture

Fatigue crack front

Sheet thickness, B

Crack extension, $\Delta c$, mm
Three-Dimensional Tunneling Simulation Analyses (ZIP3D)

Dawicke et al, 1997

\[ \Psi_c \text{ degs.} \]

\[ \text{Crack extension, } \Delta c, \text{ mm} \]

\[ \text{2024-T3 (TL)} \]
\[ B = 2.3 \text{ mm} \]

Scatter bands (surface measurements)

- \( z/B = 0 \) (mid-plane)
- \( z/B = 0.125 \)
- \( z/B = 0.25 \)
- \( z/B = 0.375 \)
- \( z/B = 0.45 \)
- \( z/B = 0.5 \) (surface)
Results of Tunneling on 2024-T351

Measured and Calculated Load against Surface Crack Extension

2024-T351 (LT)
B = 6.35 mm

○ Test (Surface), 4 specimens

Simulated $\psi_c = 5.95$ deg.
(Mid-plane; ZIP3D)
Measured and Calculated Load against Mid-Plane Crack Extension

2024-T351 (LT)
B = 6.35 mm

○ Test (Mid-plane), 4 specimens

Simulated $\psi_c = 5.95$ deg.
(Mid-plane; ZIP3D)
Effects of Various Anti-Buckling Guides on Fracture

203 mm
W = 1 m

CRACK

2024-T3
B = 1.6 mm

7075-T6
B = 12.7 mm

S

S_f = 166 MPa

206
24%

243
46%

Steel

75 mm
Comparison of Out-of-Plane Displacements

Dawicke et al, 1999

S = 240 MPa
B = 1.6 mm
c = 102 mm
w = 305 mm
y/w = 0.1

Out-of-Plane Displacement, mm

Distance from centerline (x/w)
CTOA Method Predicts the Effects of Buckling

Seshadri and Newman, 1999

![Graph showing CTOA Method Predictions](image)

- **Tests (guides)**
- **Tests (no guides)**

**2024-T3 (TL)**
- $B = 1.6$ mm
- $W = 610$ mm
- $2c_o = 203$ mm

**STAGS Analysis:**
- $\Psi_C = 5.0$ deg., $h_C = 1$ mm
NASA/FAA Wide-Stiffened Panels

W = 1016 mm

H = 2032 mm

2024-T3

7075-T6

203 mm

41 mm

Rivet hole and MSD

Leak crack

Bonded

Riveted

6 mm
CTOA Fracture Criterion Predicts Failure of Stiffened Panel with Single Crack

2024-T3 / 7075-T6
B = 1.6 / 2.2 mm
W = 1016 mm
2c_i = 203 mm
Unrestrained

Test
Stiffener failed (analysis)
Panel failed

Load, kN

STAGS
Analysis:
ψ_c = 5.4 deg.
h_c = 2 mm

Lead Crack

Crack extension, Δc, mm
Measured and Predicted Load against Crack Extension for Lead Crack and MSD

2024-T3 / 7075-T6
B = 1.6 / 2.2 mm
W = 1016 mm
2c₀ = 203 mm
Unrestrained

STAGS
Ψ_c = 5.4 deg.
h_c = 2 mm

Applied force, P (kN)

Crack

1.3-mm MSD

Stiffener

Crack extension, Δc, mm
DC-9 Aft Fuselage Section

Hsu et al, Boeing, 1999

Fuselage aft-bulkhead
Aft Pressure Bulkhead Test Article

Hsu et al, Boeing, 1999

Frame / Tee

Bulkhead Web

MSD

Lead Crack
Measured and Predicted Failure Pressure on Aft Fuselage

Hsu et al, Boeing, 1999

Predicted failure pressure = 65 kPa

Experimental Results
(62 kPa)

2014-T3 (TL)
STAGS Prediction
\( \Psi_n = 5.5 \) deg.; \( \Psi_c = 3.4 \) deg.

First MSD Linkup

Applied fuselage pressure, kPa

Crack-tip coordinates, mm
Crack-Growth Resistance Curve based on $\delta_5$ Measurements
Measured and Predicted Load-against-Crack-Opening ($\delta_5$) Displacement

James and Newman, 2003

James et al. (\(\psi_c = 6.8\) deg.)

ZIP3D (\(\psi_c = 6.8\) deg.)

WARP3D (\(\psi_c = 6.35\) deg.)

2024-T351 Compact
B = 6.35 mm
W = 152 mm
ci/W = 0.4

Crack-tip displacement, $\delta_5$, mm
Concluding Remarks

• Constraint effects, crack tunneling and crack blunting were shown to be key issues why earlier CTOA finite-element fracture analyses indicated “non-constant” values.

• Measurements and analyses on aluminum alloys for tension and bending crack configurations have supported the use of a “constant” CTOA value from initiation to failure.

• Critical CTOA and (2D Plane-strain core or 3D) finite-element fracture simulations are able to predict stable tearing and instability due to: (1) fatigue pre-cracking effects, (2) out-of-plane buckling, (3) multiple-site damage cracking and (4) an actual aircraft fuselage failure.

• A close relationship has been established between critical CTOA and $\delta_5$, but further study is needed.
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Two-Parameter Fracture Mechanics for Residual Strength

- Notch-Strength Analysis (Kuhn, 1970)
- K and T (Larrson & Carlsson, 1973; Hancock et.al., 1991)
- Two-parameter fracture criterion (TPFC; Newman, 1973)
- Two-criteria approach (Dowling and Townley, 1975)
- J and Q (Shih & O’Dowd, 1990)
- J and Stress Tri-axiality (Brocks et.al., 1995, 1998)
- K and $\alpha_g$ (Global constraint) (Newman et.al., 1995, 2000)
- J and $\alpha_h$ (Newman, 2002; Leach et.al., 2007)
Prediction of Fracture on 2219-T87 M(T) Specimens using the Critical CTOA Fracture Criterion

![Graph showing the relationship between failure stress and specimen width for 2219-T87 M(T) specimens. The graph includes test data and predictions for plane stress and plane-strain conditions. ZIP2D Analyses are also shown, with different line styles indicating plane stress, plane strain, and plane-strain core conditions. Parameters such as B = 2.54 mm, 2a_0/W = 1/3, and ψ_c = 4.7 deg., h_c = 1.9 mm are noted.]
Experimental Relationship between $K_{le}$ and $S_n$ for 2219 Aluminum Alloy Sheet Material using TPFC Analyses

Boeing tests
TPFC:
$(K_F = 210 \text{ MPa-m}^{1/2}; \ m = 0.95)$

1973 Equation ($S_n > \sigma_{YS}$)

New relation
$S_n > \sigma_{YS}$

$W = 1200$ mm
$2c_i/W = 1/3$

2219-T87
$M(T) \ B = 2.54$ mm

$\sigma_{YS}/S_u$
Experimental Relationship between $K_{le}$ and $S_n$ for 2219 Aluminum Alloy Sheet Material with CTOA Analyses

$K_{le}$ MPa-m$^{1/2}$

$S_n/S_u$ vs. $K_{le}$ plot

- Boeing tests
- TPFC: $(K_F = 210$ MPa-m$^{1/2}; m = 0.95$)
- 1973 Equation ($S_n > \sigma_{ys}$)
- ZIP2D (Plane-strain core)

$W = 1200$ mm

2219-T87
$M(T) B = 2.54$ mm
$2c_i/W = 1/3$

New relation $S_n > \sigma_{ys}$
Experimental Relationship between $K_{le}$ and Specimen Width

2219-T87
M(T) B = 2.54 mm
$2a_f/W = 1/3$

$K_F$

$K_{le}$ [MPa-m$^{1/2}$]

TPFC:
$K_F = 210$ MPa-m$^{1/2}$; $m = 0.95$

ZIP2D:
$\psi_c = 4.7$ deg.; $h_c = 1.9$ mm

Boeing tests

Width, $W$, m
Experimental Relationship between $K_{le}$ and Specimen Width

$K_F$ .................................

2219-T87
M(T) B = 2.54 mm
$2a_i/W = 1/3$

FEA & (CTOA)$_c$
(TBD?)

$K_{le}$
MPa-m$^{1/2}$

Width, W, m

TPFC:
$K_F = 210$ MPa-m$^{1/2}$; $m = 0.95$

ZIP2D:
$\psi_c = 4.7$ deg.; $h_c = 1.9$ mm

Boeing tests
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Equivalent $K_R$ (or $G_R$) Resistance Curve from TPFC

Orange (1980)

\[
\left( \frac{K_R}{K_F} \right)^2 = \frac{\Delta c}{\Delta c + (mK_F/\sigma_u)^2/\pi}
\]
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* Inspired by the works of Paul Kuhn and George Irwin.
Stress Concentration Factor for an Elliptical Hole in an Infinite Plate

Inglis (1913)

\[ K_T = 1 + 2 \frac{a}{\rho} \]

\[ \sigma_e = S K_T \]

\[ \sigma_e = S + 2 S \frac{a}{\rho} \]
Elastic-Plastic Stress- and Strain-Concentration Factors using Neuber’s Equation

Neuber (1961):

\[ K_\sigma K_\varepsilon = K_T^2 \]
\[ \sigma \ varepsilon E = \sigma_e^2 \]

Crews (1974) experimentally validated Neuber’s equation for elliptical hole in finite plate under remote uniform stress

Hutchinson (1968) showed that the stress-strain field for a crack in a non-linear elastic material verified Neuber’s equation
Fracture Conditions at Root of Very Sharp Notch (or Crack)

Newman (1973)

\[
\sigma_f \varepsilon_f E = \sigma_e^2 = (S_f K_T^*)^2
\]

\[K_T^* = 1 + 2 \sqrt{a / \rho^*}\]
Derivation of Two-Parameter Fracture Criterion for Nominal Stress Less Than Yield Stress

\[
\sigma_f \varepsilon_f E = \sigma_e^2 = (S_f K_T^*)^2 \quad K_T^* = 1 + 2 \sqrt{a / \rho^*}
\]

\[
\sqrt{\pi \sigma_f \varepsilon_f E \rho^* / 4} - S_f \sqrt{\pi \rho^* / 4} = S_f \sqrt{\pi a}
\]

\[
\sqrt{\pi \sigma_f \varepsilon_f E \rho^* / 4} \left(1 - S_f / \sqrt{\sigma_f \varepsilon_f E}\right) = K_{le}
\]

\[
K_F \left(1 - m' S_f\right) = K_{le}
\]

\[
K_F = \frac{K_{le}}{(1 - m S_f / \sigma_u)} \quad \text{for } S_f < \sigma_{ys}
\]
Two-Parameter Fracture Criterion for Nominal Stress Greater than Yield Stress

For infinite plate:

\[ S_n = S_f \]

\[
K_F = \frac{K_{le}}{\left(\sqrt{\frac{E_n}{E}} - m \frac{S_f}{\sigma_u}\right)} \quad \text{for} \quad S_f \geq \sigma_{ys}
\]
Use of Maximum Load and the Initial Crack Length to Calculate the Elastic Stress-Intensity Factor at Failure

\[ K_{le} = \frac{P_{\text{max}}}{(WB)} \sqrt{\pi a_i} \]

\[ F \]

Graph showing the relationship between load, \( P \), and crack length, \( a \), with the formula for \( K_{le} \) indicated.
Functional Form of Two-Parameter Fracture Criterion

\[ K_F = \frac{K_{le}}{\Phi} \]

\[ \Phi = 1 - m \left( \frac{S_n}{S_u} \right) \]
Fracture Locus for Thin Sheet to Thick Plate Materials

TPFC:
\[ K_F = \frac{K_{le}}{1 - m \left( \frac{S_n}{S_u} \right)} \]

Plastic-collapse stress:
\[ S_n < \sigma_{ys} \]

Elastic fracture toughness

Plane-stress region

Plane-strain plateau, \( K_{lc} \)

Thickness

Yield stress \( \sigma_{ys} \)
Fracture of 2024-T3 Thin Sheet and 7075-T651 Plate M(T) Specimens

2024-T3
B = 2.3 mm
W = 305 mm
K_f = 267 MPa-m^{1/2}
m = 1

7075-T651
B = 12.7 mm
W = 305 mm
K_f = 31 MPa-m^{1/2}
m = 0.59

S

W or 2w
Fracture of 2024-T351 Small- and Large-M(T) Specimens

(Note: All tests and analyses have $S_n > \sigma_{ys}$)
Elastic Fracture Toughness of 7075-T651 Alloy (1)

TPFC:

\[ K_F = \frac{K_{le}}{1 - m \left( \frac{S_n}{S_u} \right)} \]

\[ S_n < \sigma_{ys} \]

7075-T651  C(T)  B = 0.225 in.

\( K_{le} = 80 \text{ ksi-in}^{1/2} \)

\( m = 0.70 \)

\( \sigma_{ys} = 73 \text{ ksi} \)

\( \sigma_u = 84 \text{ ksi} \)

- Solid symbols: \( S_n > \sigma_{ys} \)
- Circles: w = 3 in. (fatigued to failure)
- Circles with open center: w = 3 in. (static to failure)
- Squares: w = 6 in. (fatigued to failure)
- Squares with open center: w = 6 in. (static to failure)

(a_i / W) vs. (Solid symbols \( S_n > \sigma_{ys} \))
Elastic Fracture Toughness of 7075-T651 Alloy (2)

7075-T651 LT (NASGRO)
B = 0.25 in.
2w = 3.0 in.

\[ K_{le} = S_f (\pi a)^{1/2} F \]

\[ \sigma_y = 77 \text{ ksi} \]
\[ \sigma_u = 85 \text{ ksi} \]

\[ K_F = 80 \text{ ksi-in}^{1/2} \]
\[ m = 0.7 \]

\[ 2w = 3 \text{ in.} \]

\[ a_i / w \]

\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]
Fracture of Hiduminium-48 Thin-Sheet Material M(T) Specimens

- Fracture toughness $K_{le}$ in MPa$\cdot$m$^{1/2}$
- Material properties:
  - $t = 3.2$ mm
  - $\sigma_{ys} = 445$ MN/m$^2$
  - $\sigma_{u} = 500$ MN/m$^2$
- Fracture toughness $K_F = 405$ MN/m$^{3/2}$
- $m = 0.95$
- Symbols:
  - $S_n > \sigma_{ys}$
  - $S_n < \sigma_{ys}$
- Graph showing $K_{le}$ vs. $2a_i/W$ for various $W$ values (200 mm, 150 mm, 125 mm, 100 mm, 75 mm, 50 mm, 25 mm).
- $S$ and $W$ or $2w$ dimensions.

Equation: $\sigma_n \leq \sigma_{ys}$. 

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Fracture of Hiduminium-48 Thin-Sheet Material C(T) Specimens

Prediction

\[ K_F = 405 \text{ MN/m}^{3/2} \]
\[ m = 0.95 \]

Eq. (7)

\[ \text{Eq. (8)} \]

\[ a_i / w = 0.5 \]

HID - 48 [14]
\[ t = 3.2 \text{ mm} \]
\[ \sigma_{ys} = 445 \text{ MN/m}^2 \]
\[ \sigma_u = 500 \text{ MN/m}^2 \]

\[ S_n > \sigma_{ys} \]
Elastic Fracture Toughness for M(T) Specimens (3)

Boeing (TL)

Ti-62222 (LT)

B = 1.7 mm

2w = 400 mm

KF = 217 MPa-m$^{1/2}$
m = 0.8

Constant amplitude

NASA LaRC (LT)

Crack length / width (a/w)

K_{le} MPa-m$^{1/2}$
Fracture of 4340 Steel C(T) Specimens

- Cyclic to failure
- Static fracture

4340 Steel C(T)
- B = 0.25 in.
- W = 2 in.

\[ \sigma_{ys} = 160 \text{ ksi} \]
\[ \sigma_u = 220 \text{ ksi} \]

TPFC:
- \( K_F = 350 \text{ ksi-in}^{1/2} \)
- \( m = 0.55 \)
Fracture of D16Cz Russian 2000-Series Aluminum Alloy
Fracture of Inconel-718 Superalloy C(T) Specimens

- Inconel 718 C(T)
- B = 9.5 mm
- w = 76.2 mm
- $\sigma_{ys} = 1070$ MPa
- $\sigma_u = 1330$ MPa

TPFC:
- $K_F = 650$ MPa$\cdot$m$^{1/2}$
- $m = 0.6$
Residual Strength Analyses of Surface Cracks

7075-T651 Aluminum Alloy

Surface cracks:
- \( t = 5 \text{ mm} \)
- \( t = 13 \text{ mm} \)

TPFC
\( K_F = 58.2 \text{ MN/m}^{3/2} \)
\( m = 0.38 \)

TPFC
\( K_F = 38.2 \text{ MN/m}^{3/2} \)
\( m = 0.09 \)

\( K_{le} = S \sqrt{\pi c} F(a,c,w,t) \)
Experimental Relationship between Two Fracture Parameters

Newman, 1973

$K_F/E$ vs $m$

$mm^{1/2}$

$m = \tanh(21 \times K_F/E)$
Summary

- Critical CTOA and finite-element analysis support the functional form of the TPFC equations, i.e., a linear relation between $K_{le}$ and failure stress.

- TPFC failure analyses are consistent with a two-parameter description of the KR-curve.

- Transferability from bend- to tension-loaded specimens has been demonstrated on a number of materials using the TPFC analyses.