8) Recent developments in the fatigue crack growth analysis

- the UniGrow fatigue crack growth model for spectrum loading
FATIGUE CRACK GROWTH ANALYSES BASED ON THE ‘UniGrow’ TWO PARAMETER FATIGUE CRACK GROWTH MODEL

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The crack is modeled as a sharp notch with finite tip radius $\rho^*$. Fatigue crack growth is regarded as successive crack increments (re-initiation) over distance $\rho^*$. The number of cycles $N^*$ necessary to break the material over the distance $\rho^*$ can be determined from the cyclic (Ramberg-Osgood) and fatigue material curve (Manson-Coffin) obtained experimentally from smooth specimens. The instantaneous fatigue crack growth rate can be determined as:

$$\frac{da}{dN} = \frac{\rho^*}{N^*}$$
Idealized discrete structure of the material and the continuum mechanics based stress field

\[ \sigma_{yy}(x) \]

\[ \rho^* \]

\[ \rho^* \]

\[ \rho^* \]

\[ \rho^* \]
The principal idea of the local stress-strain approach to fatigue life prediction method

\[ \sigma_{\text{peak}} = f(S, \text{geometry, mtl. properties}) \]

\[ \varepsilon_{\text{peak}} = g(S, \text{geometry, mtl. properties}) \]

Need to be determined!!

\[ N_a \]

\[ dN = \frac{\rho^*}{N^*} \]
Phenomena to be accounted for in the case of elastic-plastic stress-strain analysis of growing fatigue cracks

Use of appropriate SIF using Handbook or any other solution, $K_{\text{appl}}$.

Determination of the linear elastic crack tip stresses $\sigma_{\text{elast.}}$ from applied stress intensity factor $K_{\text{appl}}$ is relatively easy (Creager-Paris solution).

However, FOUR important phenomena need to be additionally accounted for while determining the elastic-plastic strains and stresses around the crack tip, i.e.

• the non-linear elastic-plastic material behaviour (Ramberg-Osgood cyclic stress-strain curve),

• material memory while applying cyclic loading (the effect of the load path or the sequence of applied loading cycles),

• the interaction between the crack tip plastic zone and the surrounding linear elastic stress field,

• the structural memory associated with the propagation of the crack tip (assessment of the length of the past stress history affecting the crack tip at the current position).
Approximate crack tip geometry, the cyclic plastic zone and the crack tip stress-strain response

A) $S_A > 0$

B) $S_A > S_B > 0$

C) $S_c = 0$

D) $S_D < 0$

Global (remote) stress history, $S$

Local crack tip stress and strains

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Geometrical/Mathematical model for linear elastic stress-strain analysis of fatigue crack

**Tension**

\[ S_{\text{max}}^+ \]

\[ \sigma_{\text{max,net}}^+ \]

\[ \rho^* \]

\[ 2a \]

\[ \frac{1}{2} \rho^* \]

\[ \sigma_{11,\text{max,net}}^+ = \frac{K_{\text{max,appl}}}{\sqrt{2\pi x}} \left(-\frac{\rho^*}{2(x - 0.5\rho^*)} + 1 \right) + \ldots \]

\[ \sigma_{22,\text{max,net}}^+ = \frac{K_{\text{max,appl}}}{\sqrt{2\pi x}} \left(\frac{\rho^*}{2(x - 0.5\rho^*)} + 1 \right) + \ldots \]

**Compression**

\[ S_{\text{min}}^- \]

\[ \sigma_{\text{min,net}}^- \]

\[ 2p^* \]

\[ 2\rho^* \]

\[ \sigma_{11,\text{min,net}}^- = \frac{K_{\text{min,appl}}}{2\sqrt{\pi a}} \cdot \frac{3\rho^*}{x^2} \left(1 - \frac{\rho^*}{x^2} \right) \]

\[ \sigma_{22,\text{min,net}}^- = \frac{K_{\text{min,appl}}}{2\sqrt{\pi a}} \left(2 + \frac{\rho^*}{x^2} + \frac{3\rho^*}{x^4} \right) \]

**Creager & Paris, valid for** \( x > \frac{1}{2} \rho^* \) !
Hypothetical linear-elastic stress field near a sharp notch (Creager-Paris solution)

1st reversal
\[ \sigma_{xx,\text{max}}(x) = \frac{K_{\text{max}}}{\sqrt{2\pi x}} \left( -\frac{\rho^*}{2x} + 1 \right) \]
\[ \sigma_{yy,\text{max}}(x) = \frac{K_{\text{max}}}{\sqrt{2\pi x}} \left( \frac{\rho^*}{2x} + 1 \right) \]
\[ \tau_{xy,\text{max}}(x) = 0 \]

2nd reversal
\[ \Delta \sigma_{xx}(x) = \frac{\Delta K}{\sqrt{2\pi x}} \left( -\frac{\rho^*}{2x} + 1 \right) \]
\[ \Delta \sigma_{yy}(x) = \frac{\Delta K}{\sqrt{2\pi x}} \left( \frac{\rho^*}{2x} + 1 \right) \]
\[ \Delta \tau_{xy}(x) = 0 \]
The INPUT: The cyclic stress-strain and fatigue strain-life curves obtained from smooth cylindrical specimens tested under cyclic strain controlled uni-axial stress state.

\[ \sigma = \frac{\sigma}{E} + \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} \]

\[ \frac{\Delta \varepsilon}{2} = \frac{\sigma^{'f}}{E} \left( 2N_f \right)^b + \varepsilon_f \left( 2N_f \right)^c \]
Calculation of the crack tip stresses and strain – the elasto-plastic analysis (the Neuber rule)

**Multiaxial stress state**

\[
\begin{align*}
\sigma_{ij,\text{net}} \varepsilon_{ij,\text{net}} &= \sigma_{ij,\text{tot}} \varepsilon_{ij,\text{tot}} \\
\varepsilon_{ij,\text{tot}} &= f(\sigma_{ij,\text{tot}})
\end{align*}
\]

**Uniaxial stress state**

\[
\begin{align*}
\sigma_{22,\text{net}} \varepsilon_{22,\text{net}} &= \sigma_{22,\text{tot}} \varepsilon_{22,\text{tot}} \\
\varepsilon_{22,\text{tot}} &= \sigma_{22,\text{tot}} \cdot \left(1 + \left(\frac{\sigma_{22,\text{tot}}}{K'}\right)^\frac{1}{n}\right)
\end{align*}
\]
**Neuber’s rule**

\[
\begin{align*}
\sigma_{22}^e \varepsilon_{22}^e &= \sigma_{22}^a \varepsilon_{22}^a \\
\varepsilon_{22}^a &= \frac{\sigma_{22}^a}{E} + f\left(\sigma_{22}^a\right)
\end{align*}
\]

**ESED method**

\[
\begin{align*}
\int_{0}^{\varepsilon_{22}^e} \sigma_{22}^e \, d\varepsilon_{22}^e &= \int_{0}^{\varepsilon_{22}^a} \sigma_{22}^a \, d\varepsilon_{22}^a \\
\varepsilon_{22}^a &= \frac{\sigma_{22}^a}{E} + f\left(\sigma_{22}^a\right)
\end{align*}
\]
Equations of the multiaxial Neuber’s rule and Hencky’s equations of plasticity

\[
\begin{align*}
\varepsilon_{11}^a(x) &= \frac{1}{E} \left( \sigma_{11}^a(x) - \nu \left[ \sigma_{22}^a(x) + \sigma_{33}^a(x) \right] \right) + \frac{f \left[ \sigma_{eq}^a(x) \right]}{\sigma_{eq}^a(x)} \left[ \sigma_{11}^a(x) - \frac{1}{2} \left[ \sigma_{22}^a(x) + \sigma_{33}^a(x) \right] \right] \\
\varepsilon_{22}^a(x) &= \frac{1}{E} \left( \sigma_{22}^a(x) - \nu \left[ \sigma_{33}^a(x) + \sigma_{11}^a(x) \right] \right) + \frac{f \left[ \sigma_{eq}^a(x) \right]}{\sigma_{eq}^a(x)} \left[ \sigma_{22}^a(x) - \frac{1}{2} \left[ \sigma_{33}^a(x) + \sigma_{11}^a(x) \right] \right] \\
\varepsilon_{33}^a(x) &= \frac{1}{E} \left( \sigma_{33}^a(x) - \nu \left[ \sigma_{11}^a(x) + \sigma_{22}^a(x) \right] \right) + \frac{f \left[ \sigma_{eq}^a(x) \right]}{\sigma_{eq}^a(x)} \left[ \sigma_{33}^a(x) - \frac{1}{2} \left[ \sigma_{11}^a(x) + \sigma_{22}^a(x) \right] \right]
\end{align*}
\]

\[
\begin{align*}
\sigma_{11}^e(x) \cdot \varepsilon_{11}^e(x) &= \sigma_{11}^a(x) \cdot \varepsilon_{11}^a(x) \\
\sigma_{22}^e(x) \cdot \varepsilon_{22}^e(x) &= \sigma_{22}^a(x) \cdot \varepsilon_{22}^a(x) \\
\sigma_{33}^e(x) \cdot \varepsilon_{33}^e(x) &= \sigma_{33}^a(x) \cdot \varepsilon_{33}^a(x)
\end{align*}
\]

where: 
\[
f \left[ \sigma_{eq}^a(x) \right] = \left[ \frac{\sigma_{eq}^a(x)}{K'} \right]^{1/n}
\]

and 
\[
\sigma_{eq}^a(x) = \frac{1}{\sqrt{2}} \sqrt{\left[ \sigma_{11}^a(x) - \sigma_{22}^a(x) \right]^2 + \left[ \sigma_{22}^a(x) - \sigma_{33}^a(x) \right]^2 + \left[ \sigma_{33}^a(x) - \sigma_{11}^a(x) \right]^2}
\]

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The hypothetical linear elastic and the actual elastic-plastic strains and stresses in the notch tip region

The hypothetical linear elastic stress field

The elastic and actual elastic-plastic distribution of the stress component $\sigma_{yy}$ in the crack/notch tip region.
The procedure for calculating $\sigma_{\text{max,tot}}$ and $\Delta\varepsilon_{\text{tot}}$ is the same as in the case of calculating the residual stress but it is carried out using the total stress intensity factors.

$$K_{\text{max, tot}} \quad \text{and} \quad \Delta K_{\text{tot}} \rightarrow \text{Neuber rule} \rightarrow \sigma_{\text{max,tot}} \quad \text{and} \quad \Delta\varepsilon_{\text{tot}}$$

Smith-Watson-Topper (SWT) parameter

$$\text{SWT} = \sigma_{\text{max,tot}} \cdot \Delta\varepsilon_{\text{tot}}/2$$

The number of cycles to failure $N_f$:

$$SWT = \sigma_{\text{max,tot}} \frac{\Delta\varepsilon_{\text{tot}}}{2} = \frac{\left(\sigma'_f\right)^2}{E} \left(2N_f\right)^{2b} + \sigma'_f \varepsilon'_f \left(2N_f\right)^{b+c} \Rightarrow N_f$$

Fatigue crack growth rate $\frac{da}{dN}$

$$\frac{da}{dN} = \frac{\rho^*}{N_f}$$
Complete set of equations for calculating fatigue crack growth rate
(constant amplitude loading)

Maximum stress and strain at the crack tip

\[
\begin{align*}
\varepsilon_{\text{max, tot}} &= \frac{\sigma_{\text{max, tot}}}{E} + \left(\frac{\sigma_{\text{max, tot}}}{K'}\right)^{1/n} \\
\end{align*}
\]

\[
\frac{1}{E} \left(\frac{K_{\text{max, tot}} \cdot x_1}{\sqrt{2\pi\rho^*}}\right)^2 = \sigma_{\text{max, tot}} \cdot \varepsilon_{\text{max, tot}}
\]

Stress and strain range at the crack tip

\[
\begin{align*}
\Delta \varepsilon_{\text{tot}} &= \Delta \sigma_{\text{tot}} \cdot \varepsilon_{\text{tot}} \\
\frac{\Delta \varepsilon_{\text{tot}}}{2} &= \frac{\Delta \sigma_{\text{tot}}}{2E} + \left(\frac{\Delta \sigma_{\text{tot}}}{2K'}\right)^{1/n} \\
\end{align*}
\]

Fatigue strain – life curve

\[
\sigma_{\text{max, tot}} \frac{\Delta \varepsilon_{\text{tot}}}{2} = \left(\frac{\sigma_f^*}{2} \right)^{2b} + \sigma_f^* \varepsilon_f^* \left(2N_f\right)^{b+c}
\]

Fatigue crack growth rate

\[
\frac{da}{dN} = \frac{\rho^*}{N_f}
\]
Calculation of the actual (total) elasto-plastic maximum stress, 
\(\sigma_{\text{max,tot}}\), and strain, \(\varepsilon_{\text{max,tot}}\), over the first element, \(\rho^*\), near the crack tip
(constant amplitude loading, analysis based on the plastic strain evolution)

Maximum stress and strain at the crack tip

\[
\frac{1}{E} \left( \frac{K_{\text{max,tot}} \cdot x_1}{\sqrt{2\pi \rho^*}} \right)^2 = \sigma_{\text{max,tot}} \cdot \varepsilon_{\text{max,tot}}
\]

\[
\varepsilon_{\text{max,tot}} = \frac{\sigma_{\text{max,tot}}}{E} + \left( \frac{\sigma_{\text{max,tot}}}{K'} \right)^{\frac{1}{n}}
\]

\[
\sigma_{\text{max,tot}} = \frac{\left( \frac{1}{n} \right)^{\frac{1}{n}} \left( x_1 \right)^2}{2\pi E \rho^*} \left( K_{\text{max,tot}}^2 \right)^{\frac{n}{n+1}}
\]

\[
\varepsilon_{\text{max,tot}} = \frac{\left( x_1 \right)^2}{2\pi E K' \rho^*} \left( K_{\text{max,tot}}^2 \right)^{\frac{1}{n+1}}
\]

Stress and strain range at the crack tip

\[
\frac{1}{E} \left( \frac{\Delta K_{\text{tot}} \cdot x_1}{\sqrt{2\pi \rho^*}} \right)^2 = \Delta \sigma_{\text{tot}} \cdot \Delta \varepsilon_{\text{tot}}
\]

\[
\frac{\Delta \varepsilon_{\text{tot}}}{2} = \frac{\Delta \sigma_{\text{tot}}}{2E} + \left( \frac{\Delta \sigma_{\text{tot}}}{2K'} \right)^{\frac{1}{n}}
\]

\[
\Delta \sigma_{\text{tot}} = \left( \frac{2K'}{4\pi E K' \rho^*} \right)^{\frac{n}{n+1}} \left( \Delta K_{\text{tot}}^2 \right)^{\frac{n}{n+1}}
\]

\[
\Delta \varepsilon_{\text{tot}} = \left( \frac{2n}{4\pi E K' \rho^*} \right)^{\frac{1}{n+1}} \left( \Delta K_{\text{tot}}^2 \right)^{\frac{1}{n+1}}
\]

Fatigue strain – life curve

\[
\sigma_{\text{max,tot}} \frac{\Delta \varepsilon_{\text{tot}}}{2} = \left( \sigma_f' \right)^2 \left( 2N_f \right)^{2b} + \sigma_f' \varepsilon_f' \left( 2N_f \right)^{b+c}
\]

\[
da = \frac{\rho^*}{N_f}
\]
Fatigue crack growth expression based on the local crack tip stress-strain history and the SWT fatigue damage parameter

\[
\frac{da}{dN} = C \left[ (\Delta K_{tot})^p \left( K_{max,tot} \right)^{1-p} \right]^\gamma = C [\Delta \kappa]^\gamma
\]

Near threshold Region I

\[
p = 0.5; \quad \gamma = -\frac{1}{b};
\]

\[
C = 2\rho^* \left( \frac{\psi_1^2}{4\pi \rho^* (\sigma_f^*)^2} \right)^{\frac{-1}{2b}}
\]

Plastic strain dominated region II

\[
p = \frac{1}{n' + 1}; \quad \gamma = -\frac{2}{b + c};
\]

\[
C = 2\rho^* \left( \frac{\psi_1^2}{\frac{n'+3}{2^{n'+1}} \pi E \rho^* \sigma_f^* \epsilon_f^*} \right)^{\frac{-1}{b+c}}
\]

\[\text{Near threshold Region I Plastic strain dominated region II}\]
The approximate analytical analysis resulted in two different crack growth expressions depending on the nature of the dominating strain at the crack tip (i.e. elastic or plastic).

Those expressions can be presented as straight lines in log-log co-ordinates!

Fatigue crack growth curves: $\frac{da}{dN}=f(\Delta\kappa)$

\begin{align*}
\Delta\kappa &= \left( K_{\text{max,tot}} \right)^{1-p} \left( \Delta K_{\text{tot}} \right)^p \\
\Delta\kappa &= \left( K_{\text{max,tot}} \right)^{0.5} \left( \Delta K_{\text{tot}} \right)^{0.5}
\end{align*}

Note that exponent $p$ varies!
Evolution of the crack tip plastic zone ahead of a stationary and fatigue crack

Stationary crack of length ‘a’ under load ‘P’

Load, P

Time, t

Crack length ‘a’

Fatigue crack of length ‘a’ under load ‘P’

Load, P

Time, t

Crack length ‘a’

Crack tip plastic zone at P=P_{\text{max}}

Envelope of plastically deformed material in the wake of a growing fatigue crack

Crack tip plastic zone at P=P_{\text{max}}
uy (COD) vs. x (for various P), R2 - unloading

Distance from the crack tip, x [mm]

COD, uy [mm]

Crack tip position

Unloading S2-S3

Pmax=3.8 kN

Pmin=0

2Pmax=7.6kN

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Hypothetical evolution of the crack tip geometry; 2\textsuperscript{nd} reversal – unloading, 3\textsuperscript{rd} reversal reloading
Stress-strain evolution near a growing crack tip
(constant amplitude loading - according to the UniGrow model)
Combination of applied and residual stress intensity factors

\[
\frac{da}{dN} = C \left[ (\Delta K_{\text{appl}} - K_r)^p \left( K_{\text{max,appl}} - K_r \right)^{1-p} \right]^{\gamma}
\]

Near threshold Region I

\[
p = 0.5; \\
\gamma = -\frac{1}{b}; \\
C = 2\rho^* \left( \frac{(x_1)^2}{4\pi\rho^* (\sigma_f')^2} \right)^{-\frac{1}{2b}}
\]

Plastic strain evolution dominated region II

\[
p = \frac{1}{n' + 1}; \\
\gamma = -\frac{2}{b + c}; \\
C = 2\rho^* \left( \frac{(x_1)^2}{\frac{n'+3}{2^{n'+1} \pi E \rho^* \sigma_f' \varepsilon_f'}} \right)^{-\frac{1}{b+c}}
\]
Crack tip displacement field and corresponding residual stress field

Plastic zone

Fatigue crack and anticipated displacement field

Plastic zone

Crack model with equivalent residual stress field

\[ \sigma_r(x) \]

\[ r_c \]

\[ 2a \]

\[ K_{\text{max, appl}} = S_{\text{max, appl}} \sqrt{\pi a Y}; \]

\[ K_{\text{min, appl}} = S_{\text{min, appl}} \sqrt{\pi a Y}; \]

\[ K_{\text{res}} = \int_{a}^{a+r_c} \sigma_r (x) m(x, a) dx \]
Calculation of the residual stress intensity factor $K_{res}$ (the weight function method) and the resultant total stress intensity factors, $K_{max,tot}$ and $\Delta K_{tot}$

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[ 1 + M_1 \left( 1 - \frac{x}{a} \right)^{\frac{1}{2}} + M_2 \left( 1 - \frac{x}{a} \right) + M_2 \left( 1 - \frac{x}{a} \right)^{\frac{3}{2}} \right]$$

$$K_r = \int_0^a \left[ \sigma_r(x) m(x,a) \right] dx$$

$$K_{max,tot} = K_{max,appl} - K_r$$

and

$$\Delta K_{tot} = \Delta K_{appl} - K_r$$
Determination of the effective crack tip radius
(from the near threshold data – predominantly elastic behavior)

\[ \Delta \sigma_{th} = \frac{\Delta K_{th} x_1}{\sqrt{2\pi \rho^*}} \quad \Rightarrow \quad \rho^* = \frac{1}{2\pi} \left( \frac{\Delta K_{th} x_1}{\Delta \sigma_{th}} \right)^2 \]
Fatigue Crack Growth, St 4340

\[ K_{\text{max,appl}}^p \Delta K_{\text{appl}}^{(1-p)} \text{ (MPa}\sqrt{\text{m}}) \]

- R=0.7 Dowling
- R=0.5 Dowling
- R=0.1 Dowling
- R=0 Dowling
- R=-0.5 Dowling
- R=-1 Dowling
- R=0.7 Taylor
- R=0.05 Taylor
- R=0.5 NASA(L)
- R=0.5 NASA(R)
- R=0 NASA(L)
- R=0 NASA(R)
- R=-1 NASA(L)
- R=-1 NASA(R)
- R=0.5 Wanhill
- R=0 Wanhill
- R=-1 Wanhill
Fatigue Crack Growth, St 4340

\[ \frac{da}{dN} = \frac{K_{\text{max}}}{\sqrt{m}} (MPa \sqrt{m}) \]

- \( R = 0.7 \) Dowling
- \( R = 0.5 \) Dowling
- \( R = 0.1 \) Dowling
- \( R = 0 \) Dowling
- \( R = -0.5 \) Dowling
- \( R = -1 \) Dowling
- \( R = 0.7 \) Taylor
- \( R = 0.05 \) Taylor
- \( R = 0.5 \) NASA(L)
- \( R = 0.5 \) NASA(R)
- \( R = 0 \) NASA(L)
- \( R = 0 \) NASA(R)
- \( R = -1 \) NASA(L)
- \( R = -1 \) NASA(R)
- \( R = 0.5 \) Wanhill
- \( R = 0 \) Wanhill
- \( R = -1 \) Wanhill
- Approx Sol'n
$da/dN$ vs. $\Delta\kappa_{pl,tot}$; Al 7075-T6 (data of Newman and Jiang, $\rho^*=5\times10^{-6}$ m)

\[ \Delta K_{tot}^{0.5} K_{\text{max,tot}}^p \text{ [MPa}\sqrt{\text{m}}]. \]
Ti 17 material

\[ y = 1 \times 10^{-22} x^{57.28} \]

\[ y = 2 \times 10^{-11} x^{7.2072} \]
Memory rules for propagating fatigue cracks

1. Only the compressive part of the minimum stress distribution affects the fatigue crack growth rate. The compressive part of the minimum stress distribution induced by the first loading cycle constitutes the minimum initial stress field used for the determination of the residual stress intensity factor.

2. If the compressive part of the minimum stress distribution induced by current loading cycle is completely inside of the previous resultant minimum stress field, the material will not “feel” it and the current minimum stress distribution should be neglected.

3. If the compressive part of the minimum stress distribution of the current loading cycle is fully or partly outside of the previous general minimum stress field they should be combined.

4. All subsequent minimum stress distributions should be considered and included into the resultant field as long as the crack tip is located inside of their compressive stress zones. In other words, when the crack tip propagates across the entire compressive stress zone of given minimum stress field it should be excluded from further analysis.
Material Stress-Strain Response at the Notch Tip Due to Variable Amplitude Cyclic Loading $F$
Memory rules (CA)

The residual stress distribution created by a loading cycle remains active as long as the propagating crack tip remains in its compressive stress zone!

Parts of stress distribution induced by both cycles (a and b) need to be accounted for!

Only part of the stress distribution induced by cycle ‘b’ should be accounted for!
Calculation of $K_r$ for constant amplitude loading

**Constant loading**

- $P$: Applied load
- $N$: Load cycle
- $\sigma_r$: Stress intensity factor
- $x$: Location
- $K_r$: Crack growth resistance

**Constant SIF**

- $K$: Stress intensity factor
- $\sigma_r$: Stress intensity factor
- $a$: Crack length
- $b$: Crack depth
- $c$: Crack width

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Fatigue crack model for calculating the applied and the residual stress intensity factor

\[ K_{\text{max,appl}} = S_{\text{max,appl}} \sqrt{\pi a Y}; \quad K_{\text{min,appl}} = S_{\text{min,appl}} \sqrt{\pi a Y} \]

\[ K_{\text{res}} = \int_{b}^{a} \sigma_r(x) m(x,a) \, dx \]
Memory rules (CA+Ovrd)

\[ K_{\text{appl}} \]

\[ \sigma_r \]

\[ x \]

\[ a \quad b \]

\[ a \quad b \]

\[ a \quad b \quad c \]

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Underload effect

Under-load may sometime eliminate the beneficial effects of all previous overloads!
Variable amplitude loading (VA)

Loading history

Residual stress profile

Residual $K_r$ profile
Material: Al 2024 – T3

Source:

Thickness \( t = 4.1 \) [mm]
Material: Al 2024-T3

\[ \frac{da}{dN} \text{ [m/cycle]} \]

\[ \Delta K_{\text{appl}} \text{ [MPa} \sqrt{\text{m}}] \]

- \( R = 0.5 \)
- \( R = 0.3 \)
- \( R = 0.1 \)
- \( R = 0 \)
Material: Al 2024-T3, $\rho^* = 3 \cdot 10^{-6}$ m

1: $\frac{da}{dN} = 2 \cdot 10^{-11} \Delta \kappa^{9.3295}$

2: $\frac{da}{dN} = 5 \cdot 10^{-10} \Delta \kappa^{3.4523}$

3: $\frac{da}{dN} = 5 \cdot 10^{-12} \Delta \kappa^{7.4869}$
Stress History ‘a’

Stress $\sigma$ [MPa]

No. of cycles
Material: Al 2024 – T3

![Graph showing crack length vs. number of cycles](chart.png)

- **Test**
- **UniGrow**

**Crack length [mm]**

- X-axis: No. of Cycles x 10^3
- Y-axis: Crack length [mm]
Stress History 'b'

No. of Cycles $\times 10^3$

Stress $\sigma$ [MPa]

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Material: Al 2024 - T3

Crack length [mm] vs. No. of Cycles x 10^3

- Test
- UniGrow
### Material 7075-T6, P3, ONR

#### Specimen Dimensions

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<th>(B1) (in.)</th>
<th>(B2) (in.)</th>
<th>(B3) (in.)</th>
<th>(B4) (in.)</th>
<th>Average 5 (in.)</th>
<th>(L) (in.)</th>
<th>(H) (in.)</th>
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<td><strong>Front Notch Length, 2a</strong></td>
<td>12.0</td>
<td>7.0</td>
<td>0.1277</td>
</tr>
<tr>
<td><strong>Back Notch Length, 2a</strong></td>
<td>12.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gripped Length</strong></td>
<td>2.5</td>
<td></td>
<td>0.1276</td>
</tr>
</tbody>
</table>

| Nominal BGL & BGU (in.) | 0.15 | AVERAGE SECTION AREA (in²) | 0.3273 |

#### Precracking Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Precrack Stress (ksi)</td>
<td>14</td>
</tr>
<tr>
<td>Min. Precrack Stress (ksi)</td>
<td>1.4</td>
</tr>
<tr>
<td>Max. Precrack Load (lb)</td>
<td>4593</td>
</tr>
<tr>
<td>Min. Precrack Load (lb)</td>
<td>458</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>10</td>
</tr>
<tr>
<td>WaveShape</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>R Ratio</td>
<td>0.1</td>
</tr>
<tr>
<td>Mean Load (lb)</td>
<td>2521</td>
</tr>
<tr>
<td>Load Range (lb)</td>
<td>4126</td>
</tr>
<tr>
<td>Precrack Target 2a (in.)</td>
<td>0.20</td>
</tr>
<tr>
<td>Max. Precrack 2a (in.)</td>
<td>0.24</td>
</tr>
</tbody>
</table>

#### Spectrum Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Spectrum Stress (ksi)</td>
<td>23.973</td>
</tr>
<tr>
<td>Max. Spectrum Load (lb)</td>
<td>7847</td>
</tr>
<tr>
<td>Max. Load Rate (kps/s)</td>
<td>25</td>
</tr>
<tr>
<td>Min./Max. Frequency (Hz)</td>
<td>15/15</td>
</tr>
<tr>
<td>Waveshape</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>Level Multiplier (lb)</td>
<td>7847</td>
</tr>
<tr>
<td>Level Reference</td>
<td>0</td>
</tr>
<tr>
<td>Segments</td>
<td>1,315,900</td>
</tr>
<tr>
<td>Anti-Buckling (AB) Guides</td>
<td>Yes</td>
</tr>
<tr>
<td>AB Guide Torque (lb-in.)</td>
<td>40</td>
</tr>
</tbody>
</table>

#### Potential Drop System Information

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Current (amps)</td>
<td>24.0 (0.6 V)</td>
</tr>
<tr>
<td>POT. Lead Gage Length (in.)</td>
<td>0.133</td>
</tr>
<tr>
<td>Initial Ref. Voltage (µV)</td>
<td>49,943</td>
</tr>
<tr>
<td>Initial Measured &amp; Corrected Voltage (µV)</td>
<td>34,397</td>
</tr>
</tbody>
</table>

#### Environmental Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>RT</td>
</tr>
<tr>
<td>Relative Humidity</td>
<td>Ambient</td>
</tr>
</tbody>
</table>

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P3 aircraft stress history

Smax = 31.802 ksi, Smin = -6.717 ksi
FCA 301, 50th Percentile
Tension Dominated Spectrum

Max Stress: 29,755 psi
Min Stress: -6,190 psi
#EventPoints in Block: 584,263

Half Crack Length [in]

No. of Cycles

Test3-CouponData
Test2-CouponData
UniGrow prediction

Max Stress: 29,755 psi
Min Stress: -6,190 psi
#EventPoints in Block: 584,263
UK Round Robin Program, Al 7010

Initial Flaw Location
2mm radius corner crack

All dimensions are in mm

SECTION BB
6 Holes Drill + Ream
20.0 +0.10/-0.00

All dimensions are in mm

2BA Dia Thread
Typ 4 Places

Section BB
on next page

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\[ K_{A_i} = m_{A_i}(x, y; P) = \frac{P\sqrt{2}}{\pi \rho_{PjA_i}^2} \times \frac{\sqrt{\Gamma_c + \Gamma_b}}{\Gamma_c} \]
Stress Intensity Factor - the geometric correction factor $'\beta'$

- Round Robin 'Beta' from Irving (FE)
- BEASY 'Beta' from Newman (BE)
- Resultant 'Beta' from Glinka (WF)
Material: Al 7010-T73651, UK Round Robin Test

![Graph showing the relationship between ΔK_appl (MPa√m) and da/dN (m/cycle) for different R values: R=0.9, R=0.7, R=0.4, R=0.1.](image)
Material: Al 7010 -T73651, UK Round Robin Test, \( \rho^* = 2.6 \times 10^{-6} \text{m} \)

\[ y = 1 \times 10^{-9} x^{2.9065} \]
\[ y = 9 \times 10^{-13} x^{7.4114} \]
\[ y = 6 \times 10^{-11} x^{4.0382} \]
\[ y = 2 \times 10^{-13} x^{12.294} \]

\( \Delta \kappa = \Delta K_{\text{tot}} (1-p) K_{\text{max, tot}}^p \) [MPa√m]
Astrix spectrum loading, UK Round Robin Test

Number of reversals

Normalized appl. stress
Material: Al 7010-T73651, UK Round Robin Test

- Test 1
- Test 2
- UniGrow

Crack size $a$ [mm] vs. Flight hours [hrs]

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Summary of the Procedure

Advantages

• Accurate master FCG curve can be generated from limited FCG data,

• Any variable amplitude stress spectra can be analyzed,

• No prior information concerning FCG under variable amplitude loading required,

• Wide variety of real crack/load configurations can be analyzed via applications of the weight function,

• Enables modeling of irregular planar cracks

Disadvantages

• Required cyclic stress-strain smooth specimen data

• Required limited CA FCG data

• Requires sometime the use of catalogue SIF solutions or FE SIF data