Analysis Approaches for Fatigue of Rubber

W. V. Mars
Cooper Tire & Rubber Co.

UTMIS
25 October 2006
Borås, Sweden
Outline

• Overview
  – Motivation
  – Complexity
    • Loading
    • Material Behavior
• Historical Perspective
• Phenomenological Perspective
• Nucleation Approach
  – Equivalence Parameters
• Fracture Mechanics Approach
  – Energy Release Rate
• Crack Nucleation vs. Crack Growth
  – Flaws in rubber
  – When is a crack small?
Developing a Rubber Component
Complexity in Loading

Applied to Structure

Material Characterization

Multiaxial

Variable Amplitude

Experienced by Crack
Complexity in Material Behavior

- Stress-Strain
  - Finite Strains
  - Nonlinear Elasticity
  - Mullins Effect
- Crack Growth
  - Cyclic vs. Time-dependent
  - Strength
  - Threshold
  - R-ratio
  - Ozone / Chemical Attack
  - Ageing
"Many theories can be devised to explain the way the dynamic fatigue properties of rubber vary with the strain and the strain oscillation conditions, but such theories are still conjectures".

S. M. Cadwell, R. A. Merrill, C. M. Sloman, F. L. Yost
History of Rubber Fatigue Theories

1940: Cadwell, Merrill, Sloman, Yost
- Tearing Energy as a Strength Criterion

1950

1960: Identification of Ozone Effects on crack growth
- Tearing Energy as a Fatigue Crack Growth Parameter
- Crack “Nucleation” rationalized in terms of Growth of Pre-existing Flaws

1970: Regimes of Fatigue Crack Growth Identified
- Crack Growth under non-relaxing conditions

1980: Computational procedures for evaluation of Tearing Energy

1990: Application of FEA to Evaluate Tearing Energy in Complex Situations
- Nucleation of small cracks under complex load histories

2000
Typical Fatigue Failure Process

Crack Size vs. Applied Cycles

- Nucleation
- Growth

Crack Size:
- $a_0$
- $a_f$

Applied Cycles:
- $N_f$
Nucleation Approaches

- In the crack nucleation approach, we consider that, at a given level of mechanical severity, the material has a corresponding fatigue life. The relationship between the mechanical severity and the fatigue life is considered to be a characteristic of the material.

- Maximum Principal Stress
- Maximum Principal Strain
- Strain Energy Density
- Octahedral Shear Strain

Lake and Lindley, 1965
Nucleation Approaches
Equivalence Parameters

- Cracking Energy Density
- Maximum Principal Strain
- Strain Energy Density
- Octahedral Shear Strain
Parameter Correlation

\[ N_f = 16100 \left( W_{c,\text{max}} \right)^{-2.15} \]
\[ r^2 = 0.78 \]

\[ N_f = 25800 \left( W_{\max} \right)^{-2.12} \]
\[ r^2 = 0.70 \]

\[ N_f = 10800 \left( \varepsilon_{1,\text{max}} \right)^{-3.52} \]
\[ r^2 = 0.83 \]
The Ideal Equivalence Parameter

• Physical Meaning – Should it work?
  – Ability to make reasonable predictions outside the range of experimental calibration / validation
  – Connection to Fracture Mechanics
  – Quantifies experience of the critical cracking plane
• Robustness – Does it always work?
  – Correlation of nucleation life from different modes of deformation
  – Tension and Compression
• Accessibility – How practical to use?
  – User friendliness
  – Computational efficiency
# Comparison of Available Parameters

<table>
<thead>
<tr>
<th>Scalar Parameters</th>
<th>Physical Meaning</th>
<th>Robustness</th>
<th>Accessibility</th>
<th>WVM Recommendation</th>
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</table>
| Strain Energy Density | • Not all stored energy is available for release on a given plane  
• Connection to FM for ST loading, but not other cases  
• No reference to critical plane | • Worst correlation across range of deformation states | • Easily estimated from SED function | • Do Not Use |
| Max Principal Strain | • For many load histories, cracks occur on plane experiencing greatest tensile strain | • Good correlation across range of deformation states | • Easily calculated from experiment or FEA | • Useful for simple load histories |
| Max Principal Stress | • For many load histories, cracks occur on plane experiencing greatest tensile stress | • Santier and others have successfully applied this parameter over a range of deformation states | • Easily calculated from FEA | • Useful for simple load histories |
| Plane-Specific Parameters | | | | |
| Cracking Energy Density | • Energy density available on specific material plane  
• Can identify cracking plane in cases involving compression loading | • Good correlation across range of deformation states | • Requires expensive calculation to perform numerical integration / search for critical plane | • Can be improved on, but probably provides truest picture yet when loading is very complex |
The Crack Growth Approach

“...the weakness of isotropic solids, as ordinarily met with, is due to the presence of discontinuities, or flaws, as they may be more correctly called, whose ruling dimensions are large compared with molecular distances.”

– A. A. Griffith, 1920
Why Fracture Mechanics?

Cracks are everywhere.

When, Not If!
Lab crack = Real World crack?

When will the crack tip in the lab test experience the same conditions as the crack tip in the real world?
Griffith’s Approach

\[
d\frac{\Pi}{dA} = d\frac{(U - V)}{dA} + \frac{dS}{dA} = 0
\]

- Conservation of Energy
- Energy Lost Mechanically shows up as work required to extend crack
- Crack Driving Force = Rate of Energy Drop in Structure, per unit increase in crack area

(Published in 1920!)
Energy Release Rate

• AKA
  – Crack Driving Force
  – Tearing Energy
  – J-Integral

• Units
  – J/m^2
  – N/m

• A measure of crack tip loading
• Can be estimated from ‘macro’ measurements

\[ T = G = J = - \frac{d(U - V)}{dA} \]
Rivlin and Thomas

\[ T_1 = f(\text{SED}_{\text{tin}}) \]

\[ T_2 = -\frac{dU_{\text{global}}}{dA} \]

Rupture of Rubber Series - 1953 - 1964
Modern Application

Lake, Thomas, Gent, Lindley, MRPRA/TARRC, MERL
Crack nucleation and growth processes are governed by the same underlying behavior.


\[ c_0 \approx 25 \times 10^{-6} \text{ m} \]
Nucleation Deserves Special Attention Because…

- Nucleation = Growth of small cracks
- Small cracks occur frequently
- Identify failure location
- Simplify estimation of crack driving forces

- Multiaxial Loading?
- What is small?
When is a crack small?

Energy Release Rate

Crack Size

“Small”

- ERR scales with size
- Nucleation approach can be applied effectively here

“Large”

- Calls for case-specific Fracture Mechanics analysis

Diagram:

- Graph with x-axis labeled “Crack Size” and y-axis labeled “Energy Release Rate”
- Interactive areas indicating “Small” and “Large” regions
- Notes on the graph: "Energy Release Rate (ERR) scales with size. Nucleation approach can be applied effectively here. Calls for case-specific Fracture Mechanics analysis."
Summary

• Rubber Fatigue broadly resembles Metal Fatigue
  – Both Nucleation and Propagation approaches apply
  – Both approaches provide a framework for material characterization, evaluation of loading, prediction of fatigue life
  – Propagation / Fracture Mechanics approach is mature
  – Nucleation is governed by FM principles, but deserves special attention, and is less mature
    • Max Prin. Stress or Max Prin. Strain
    • Scalar vs. Plane-Specific approaches
  – Nucleation approach applies when energy release rate scales with crack size

• Rubber Fatigue differs from Metal Fatigue in a large number of important details
  – Kinematics
  – Stress-Strain Behavior
  – Fatigue phenomenology